DOA Estimation Based on Propagator Method Algorithm

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Keywords: Propagator Method Algorithm; DOA; Direction of Arrival

Abstract: This paper studies PM algorithm in direction of arrival(DOA) estimation of signals, explains the principle of this algorithm, conducts simulation for uniform linear array with Matlab, concludes that PM algorithm has different performances in estimating DOA with different degrees between angles, different SNRs and different snapshots. With the comparison of PM with Capon, Music and Esprit algorithms, the advantages and disadvantages of PM algorithm are examined. The PM algorithm has fast calculation speed and small amount of calculation. In the scenarios of high SNRs, PM algorithm has high estimation precision and stability.

1. Introduction

Most direction of arrival (DOA) estimation algorithms which based on decomposition of the subspace need to decompose the covariance matrix or data matrix by EVD(Eigen Value Decomposition) or SVD(Singular Value Decomposition). With the increase of the element number, the amount of calculation gets an exponential increase. Propagator Method (PM) algorithm[1], proposed by Marcos, is able to reduce the amount of calculation. PM algorithm obtains the noise subspace by linear calculation instead of EVD or SVD, so the amount of calculation is less than other subspace decomposition DOA estimation algorithms. This paper examines the characteristics of PM algorithmin in different scenarioss, compares it with some classical DOA estimation algorithms such as Capon[2], Music[3], Esprit[4]. The simulations are done with Matlab.

2. Propagator Method algorithm

Suppose there are D source signals impinge to uniform line array with M elements. The signal direction is $\theta_1\ \theta_2\ \ldots\ \theta_D$. $A=[a(\theta_1),\cdots a(\theta_D)]$ is $M\times D$ -dimensional matrix of the steering vector $a(\theta_i)$, in which $a(\theta_i)=[1,e^{-j\omega_i},\cdots e^{-j(M-1)\omega_i}]^T$ and $i=1,2...D$. $S(k)=[s_1(k),\cdots x_D(k)]^T$ is the D-dimensional vector of the complex amplitude of the source signals and N is the noise vector. The received signals can be described as

$$X = AS + N \tag{1}$$

The matrix of the steering vectoris A with full rank in column and there are D linear independent lines. A can be decomposed into two parts according to

$$A=\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}\begin{matrix} \} \\ \} \end{matrix} \qquad \begin{matrix} D \\ M-D \end{matrix} \tag{2}$$

where A_1 is $D\times D$ -dimensional matrix and A_2 is $(M-D)\times D$ -dimensional matrix.

The D lines in A_1 are linear independent, and A_2 can be obtained from the linear transformation of A_1 by (3),

$$P^H A_1 = A_2 \tag{3}$$

where P is defined as propagator and P^H is the transpose matrix of P . Let

  DOI: 10.25236/iiicec.2019.070

$$Q^H = [P^H, -I_{M-D}] \tag{4}$$

Then

$$Q^H A = [P^H, -I_{M-D}] \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0_{(M-D)\times D} \tag{5}$$

where I_{M-D} is $(M-D)\times(M-D)$ -dimensional unit matrix.

Divide the data matrix X into two parts:

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \begin{matrix} \} & \text{D} \\ \} & \text{M-D} \end{matrix} \tag{6}$$

When cost function $\left\| X_2 - \hat{P}^H X_1 \right\|_F^2$ gets the theoretical minimum value, the estimation of $\hat{P}$ achieves the best value by (7)

$$\hat{P} = (X_1 X_1^H)^{-1} X_1 X_2^H \tag{7}$$

Another method to get the best estimation value of $\hat{P}$ is by the partition of the covariance matrix

$$R = E[XX^H] = [G_{M\times D} \quad H_{M\times(M-D)}] \tag{8}$$

When cost function $\left\| H - G\hat{P} \right\|_F^2$ has the theoretical minimum, the estimation of $\hat{P}$ achieves the best value by (9)

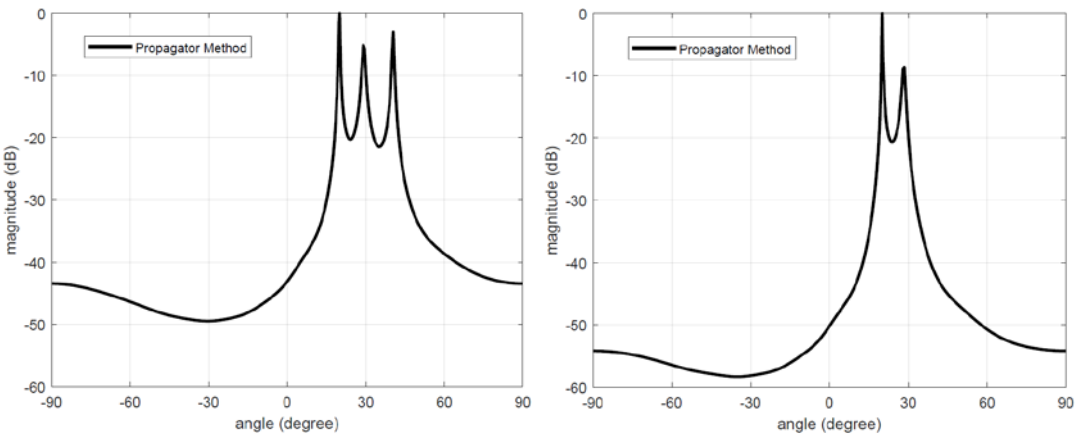
$$\hat{P} = (GG^H)^{-1} G^H H \tag{9}$$

then matrix $\hat{Q}$ which is orthogonal to A can be obtained, and the space spectrum can be obtained by (10)

$$p(\theta) = \frac{1}{a^H(\theta)\hat{Q}\hat{Q}^H a(\theta)} \tag{10}$$

3. Simulation and comparison

3.1 DOA estimation performance of different degrees between angles



(a) degrees between angles are 10° (b) degrees between angles are 5°

Fig.1. Estimation performance of different degrees between angles

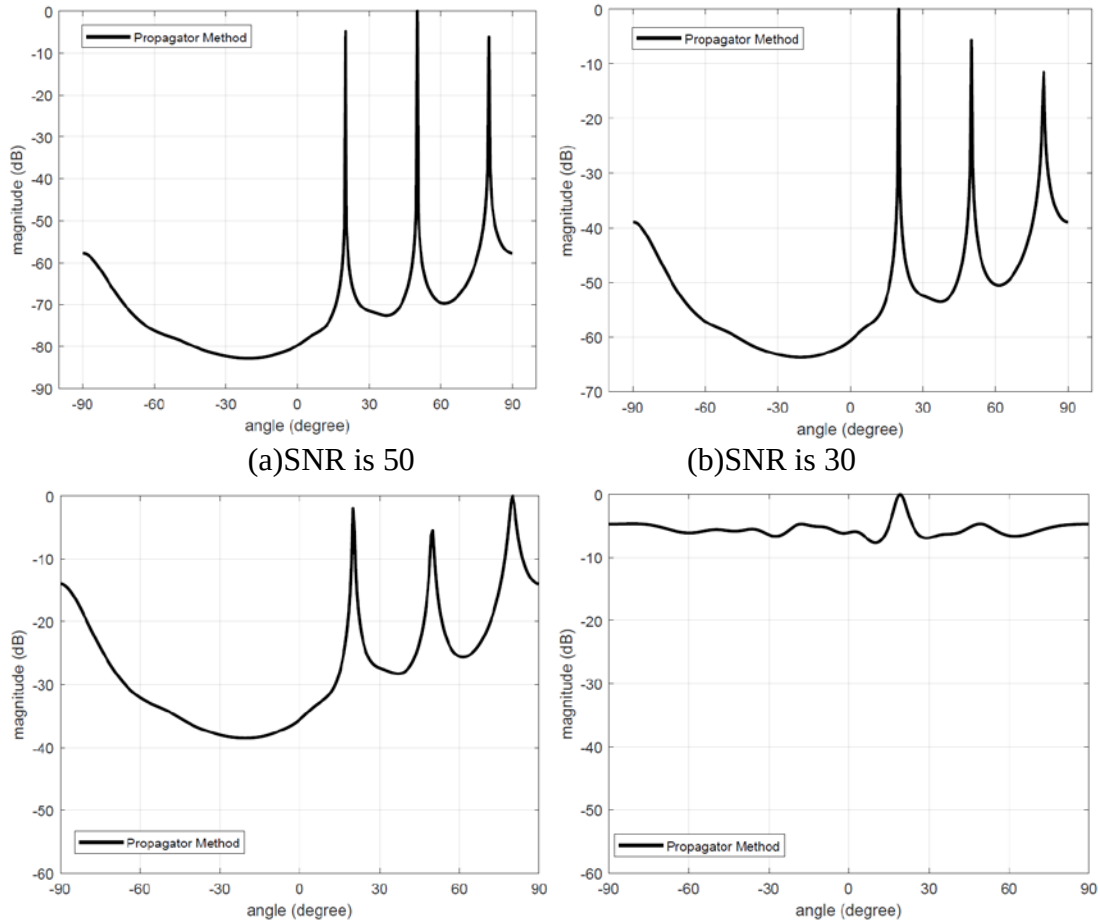
There are 3 separate narrowband signal sources impinge to the linear array. The number of array elements is 10 and the distance between two adjacent array elements is 0.5 meters. The snapshot is

200 and the signal-to-noise ratio is 20. Two experiments are done with different degrees between angles, which are 10° and 5° respectively. One scenario is with 20 degrees, 30 degrees, 40 degrees, and the other is with 20 degrees, 25 degrees, 30 degrees. The results are shown in figure 1.

Figure 1 shows that when the degrees between angles of source signals are big, PM algorithm is able to estimate the DOA of the source signals. With the decrease of degrees between the angles, the resolution of PM algorithm becomes worse.

3.2 DOA estimation performance of different signal-to-noise ratio

There are 3 separate narrowband signal sources impinge to the linear array with 20 degrees, 50 degrees, 80 degrees in the experiments. The number of array elements is 10 and the distance between two adjacent array elements is 0.5 meters. The snapshot is 200. The experiments are done under the condition that signal-to-noise ratio are 50, 30, 10 and -10 respectively, and the results are shown in figure 3.



(a)SNR is 50 (b)SNR is 30

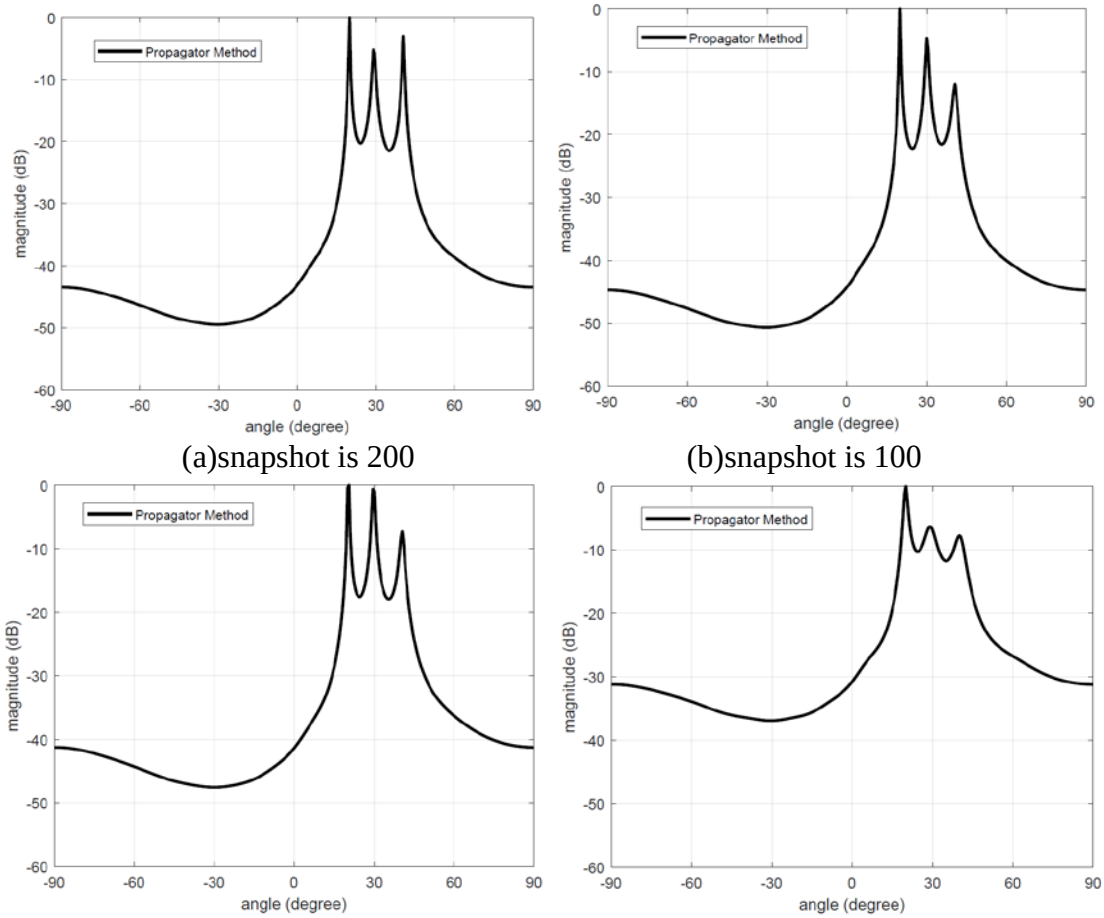
(c)SNR is 10 (d)SNR is -10

Fig.2. Estimation performance of different signal-to-noise ratio (SNR)

Figure 2 shows that when the signal-to-noise ratio of source signals are big, PM algorithm is able to estimate the DOA of the source signals. With the decrease of signal-to-noise ratio, the resolution of PM algorithm becomes worse. The algorithm may be unable to get the right DOA of each source signals in the scenarios of low SNRs.

3.3 DOA estimation performance of different snapshots

There are 3 separate narrowband signal sources with 20 degrees, 30 degrees, 40 degrees impinge to the linear array in the experiments. The number of array elements is 10 and the distance between two adjacent array elements is 0.5 meters. The snapshot is 200 and the signal-to-noise ratio are 20. The experiments are done with snapshot of 200, 100, 50 and 20. The results are shown in figure 3.



(a)snapshot is 200 (b)snapshot is 100

(c)snapshot is 50 (d)snapshot is 20

Fig.3. Estimation performance of different snapshots

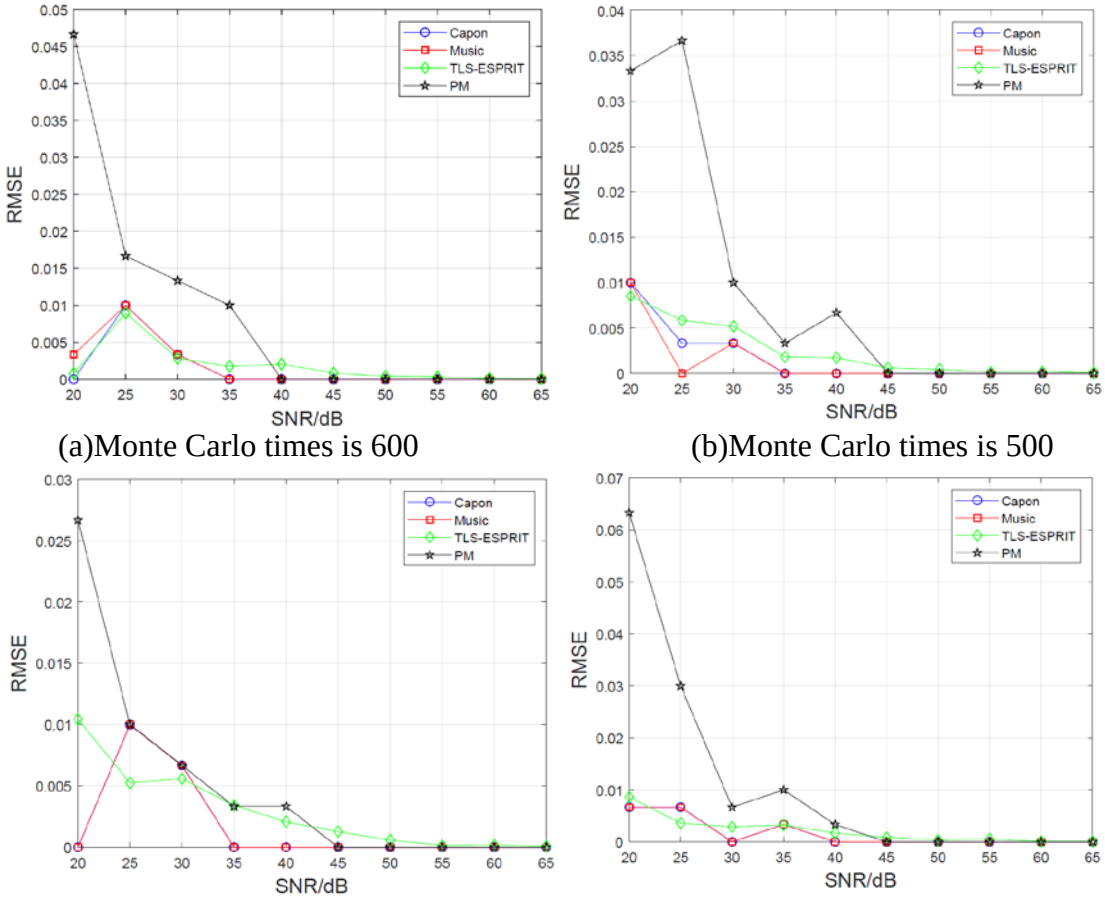
From figure 3, it is explicitly that the performance of DOA estimation has some relation with snapshots. With higher value of snapshot, the resolution of DOA estimation of PM algorithm outweighs that with lower value of snapshot. With the decrease of snapshots, the resolution of PM algorithm becomes worse. But higher value of snapshot leads to the increase of amount of calculation. It is wisely to choose a proper snapshot and get a better trade-off between the value of snapshot and the calculation cost.

4. Comparition of PM algorithm and other algorithms

Propagator method algorithm decreases the amount of calculation by using linear operation. It outweighs Capon, Music, Esprit algorithms on calculation time. Next we consider the limitation of the PM algorithm from the RMSE point of view.

Suppose there are 3 signals impinge into uniform line array with 10 elements. The distance between two elements is 0.5 meters, and the snapshot is 1000. The direction of arrival of the signals are 20°, 40°, 60°. The estimation of DOA is conducted by searching the peak of spectral function. And the RMSE of PM, Capon, Music and Esprit algorithms are calculated with Monte Carlo method.

As Figure 4 shows, PM algorithm can achieve nearly unbiased estimations if the value of signal-to-ratio is high. But in the low signal-to-ratio scenarios, the performance of RMSE of Capon, Music algorithm outweighs that of PM. The Monte Carlo method can be very time consuming. The calculation time is mainly determined by the number of Monte Carlo times. In order to get relatively precise result, it is recommended that the number is set no less than 300 in our experiments.



(a)Monte Carlo times is 600 (b)Monte Carlo times is 500

(c)Monte Carlo times is 400 (d)Monte Carlo times is 300

Fig.4. RMSE vs SNR of different algorithms with different Monte Carlo times

5. Conclusion

This paper analyses the principle of PM algorithm, compares the DOA estimation performance with different degrees between angles, different SNRs and different snapshots. By comparing PM algorithm with other three classic algorithms, Capon, Music and Esprit algorithm, from the RMSE, the advantages and disadvantages of PM algorithm are comprehensively examined. In practical applications, the PM algorithm has fast calculation speed and need small amount of calculation, but its performance is poor in the scenarios of low SNRs, small snapshots and small degrees between angles. Experimental results show that with the increase of degrees between angles, SNRs, snapshots, PM algorithm for DOA estimation has higher resolution. Under certain conditions the algorithm has high estimation precision and stability.

Acknowledgement

The research of this paper was sponsored by State Key Laboratory of Millimeter Waves of Southeast University (Project No. K201825) and Jiangsu Overseas Visiting Scholar Program for University Prominent Young & Middle-aged Teachers and Presidents(2018).

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